Fokker-Planck Equation

$$\frac{Df^\alpha}{Dt} \equiv \frac{\partial f^\alpha}{\partial t} + \mathbf{v}\cdot\nabla f^\alpha + \mathbf{F}\cdot\nabla_{\mathbf{v}} f^\alpha = \left(\frac{\partial f^\alpha}{\partial t}\right)_{\text{coll}},$$

where $\mathbf{F}$ is an external force field. The general form of the collision integral is $(\partial f^\alpha/\partial t)_{\text{coll}} = -\sum_\beta \nabla_{\mathbf{v}}\cdot\mathbf{J}^{\alpha\backslash\beta}$, with

$$\mathbf{J}^{\alpha\backslash\beta} = 2\pi\lambda_{\alpha\beta}\frac{e_\alpha^2 e_\beta^2}{m_\alpha}\int d^3v'(u^2\boldsymbol{I} - \mathbf{uu})u^{-3} \cdot\left\{\frac{1}{m_\beta}f^\alpha(\mathbf{v})\nabla_{\mathbf{v}'}f^\beta(\mathbf{v}') - \frac{1}{m_\alpha}f^\beta(\mathbf{v}')\nabla_{\mathbf{v}}f^\alpha(\mathbf{v})\right\},$$

(Landau form) where $\mathbf{u} = \mathbf{v}' - \mathbf{v}$ and $\boldsymbol{I}$ is the unit dyad, or alternatively,

$$\mathbf{J}^{\alpha\backslash\beta} = 4\pi\lambda_{\alpha\beta}\frac{e_\alpha^2 e_\beta^2}{m_\alpha^2}\left\{f^\alpha(\mathbf{v})\nabla_{\mathbf{v}}H(\mathbf{v}) - \frac{1}{2}\nabla_{\mathbf{v}}\cdot[f^\alpha(\mathbf{v})\nabla_{\mathbf{v}}\nabla_{\mathbf{v}}G(\mathbf{v})]\right\},$$

where the Rosenbluth potentials are

$$G(\mathbf{v}) = \int f^\beta(\mathbf{v}')u\,d^3v',$$

$$H(\mathbf{v}) = \left(1 + \frac{m_\alpha}{m_\beta}\right)\int f^\beta(\mathbf{v}')u^{-1}d^3v'.$$

If species α is a weak beam (number and energy density small compared with background) streaming through a Maxwellian plasma, then

$$\mathbf{J}^{\alpha\backslash\beta} = -\frac{m_\alpha}{m_\alpha + m_\beta}\nu_s^{\alpha\backslash\beta}\mathbf{v}f^\alpha - \frac{1}{2}\nu_\parallel^{\alpha\backslash\beta}\mathbf{vv}\cdot\nabla_{\mathbf{v}}f^\alpha - \frac{1}{4}\nu_\perp^{\alpha\backslash\beta}\left(v^2\boldsymbol{I} - \mathbf{vv}\right)\cdot\nabla_{\mathbf{v}}f^\alpha.$$

B-G-K Collision Operator

For distribution functions with no large gradients in velocity space, the Fokker-Planck collision terms can be approximated according to

$$\frac{Df_e}{Dt} = \nu_{ee}(F_e - f_e) + \nu_{ei}(\bar{F}_e - f_e);$$

$$\frac{Df_i}{Dt} = \nu_{ie}(\bar{F}_i - f_i) + \nu_{ii}(F_i - f_i).$$

The respective slowing-down rates $\nu_s^{\alpha\backslash\beta}$ given in the Relaxation Rate section above can be used for $\nu_{\alpha\beta}$, assuming slow ions and fast electrons, with ϵ re-