Coefficients in Riemann solver (Derivation of Eq 11–15)

(See Eq (9) in notes)

$$\alpha_3 = \frac{b}{2c^2}\left[(2h - 2q^2)\,\delta_0 + 2u\,\delta_1 + 2v\,\delta_2 + 2w\,\delta_3 - 2\,\delta_4 \right]$$

$$\alpha_3 = \frac{b}{c^2}\left[(h - q^2)\,\delta_0 + u\,\delta_1 + v\,\delta_2 + w\,\delta_3 - \delta_4 \right] \qquad (11)$$

$$\alpha_1 = \frac{\cancel{b}}{\cancel{2c^2}}\left[-\frac{\cancel{2}v\cancel{c^2}}{\cancel{b}}\,\delta_0 + \frac{\cancel{2c^2}}{\cancel{b}}\,\delta_2 \right]$$

$$\alpha_1 = -v\,\delta_0 + \delta_2 \qquad (12)$$

$$\alpha_2 = \frac{\cancel{b}}{\cancel{2c^2}}\left[-\frac{\cancel{2}w\cancel{c^2}}{\cancel{b}}\,\delta_0 + \frac{\cancel{2c^2}}{\cancel{b}}\,\delta_3 \right]$$

$$\alpha_2 = -w\,\delta_0 + \delta_3 \qquad (13)$$

$$\alpha_4 = \frac{b}{2c^2}\left[\left(\frac{q^2}{2} - \frac{uc}{b}\right)\delta_0 + \left(-u + \frac{c}{b}\right)\delta_1 - v\,\delta_2 - w\,\delta_3 + \delta_4 \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2} - \frac{uc}{b}\right)\delta_0 - \underbrace{(u\,\delta_1 + v\,\delta_2 + w\,\delta_3 - \delta_4)}_{\frac{c^2}{b}\alpha_3 - (h - q^2)\,\delta_0} + \frac{c}{b}\,\delta_1 \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2} - \frac{uc}{b}\right)\delta_0 - \left(\frac{c^2}{b}\alpha_3 - (h - q^2)\,\delta_0\right) + \frac{c}{b}\,\delta_1 \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2} - \frac{uc}{b} + h - q^2\right)\delta_0 - \frac{c^2}{b}\alpha_3 + \frac{c}{b}\,\delta_1 \right]$$

$$= \frac{\cancel{b}}{2c}\left[\left(-\cancel{\frac{q^2}{2}} - \frac{u\cancel{c}}{\cancel{b}} + \frac{c^{\cancel{2}}}{\cancel{b}} + \cancel{\frac{q^2}{2}}\right)\delta_0 - \frac{c^{\cancel{2}}}{\cancel{b}}\alpha_3 + \frac{\cancel{c}}{\cancel{b}}\,\delta_1 \right]$$

$$= \frac{1}{2c}\left[(c - u)\,\delta_0 - c\,\alpha_3 + \delta_1 \right] \qquad (14)$$

$$\alpha_0 = \frac{b}{2c^2}\left[\left(\frac{q^2}{2}+\frac{uc}{b}\right)\delta_0 + \left(-u-\frac{c}{b}\right)\delta_1 - v\,\delta_2 - w\,\delta_3 + \delta_4 \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2}+\frac{uc}{b}\right)\delta_0 - \frac{c}{b}\delta_1 - \underbrace{\left(u\delta_1 + v\delta_2 + w\delta_3 - \delta_4\right)}_{\frac{c^2}{b}\alpha_3 - (h-q^2)\delta_0} \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2}+\frac{uc}{b}\right)\delta_0 - \frac{c}{b}\delta_1 - \left(\frac{c^2}{b}\alpha_3 - (h-q^2)\delta_0\right) \right]$$

$$= \frac{b}{2c^2}\left[\left(\frac{q^2}{2}+\frac{uc}{b} + h - q^2\right)\delta_0 - \frac{c}{b}\delta_1 - \frac{c^2}{b}\alpha_3 \right]$$

$$= \frac{b}{2c^2}\left[\left(\cancel{-\frac{q^2}{2}}+\frac{uc}{b} + \frac{c^2}{b} \cancel{+\frac{q^2}{2}}\right)\delta_0 - \frac{c}{b}\delta_1 - \frac{c^2}{b}\alpha_3 \right]$$

$$\cancel{\alpha_0 = \frac{u}{2c}\delta_0 + \frac{1}{2}\delta_0 - \frac{1}{2c}\delta_1 - \frac{1}{2}\alpha_3}$$

$$\alpha_0 = \frac{1}{2c}\left[(u+c)\delta_0 - c\alpha_3 - \delta_1 \right]$$

also

$$\alpha_4 = \frac{1}{2c}\left[(c-u)\delta_0 - c\alpha_3 + \delta_1 \right]$$

add

$$\alpha_0 + \alpha_4 = \frac{1}{2c}\left[\cancel{u\delta_0} + c\delta_0 + c\delta_0 \cancel{- u\delta_0} - 2c\alpha_3 \right]$$

$$= \frac{1}{2c}\left[2c\,\delta_0 - 2c\alpha_3 \right] = \delta_0 - \alpha_3$$

or $$\alpha_0 = \delta_0 - \alpha_3 - \alpha_4 \qquad (15)$$